Marks : 40	SYJC FEB' 19 Subject : Maths – II Linear Inequalities & Bivariate Frequency	Duration : 1.5 Hours. Set – A SOLUTION

Q.1. Attempt any Two : (2 Marks each) **(04)**

1. $x - y \geq 0$

Consider the equation 4x + 5y = 40.

To draw the graph of this equation, two points are obtained as follows :

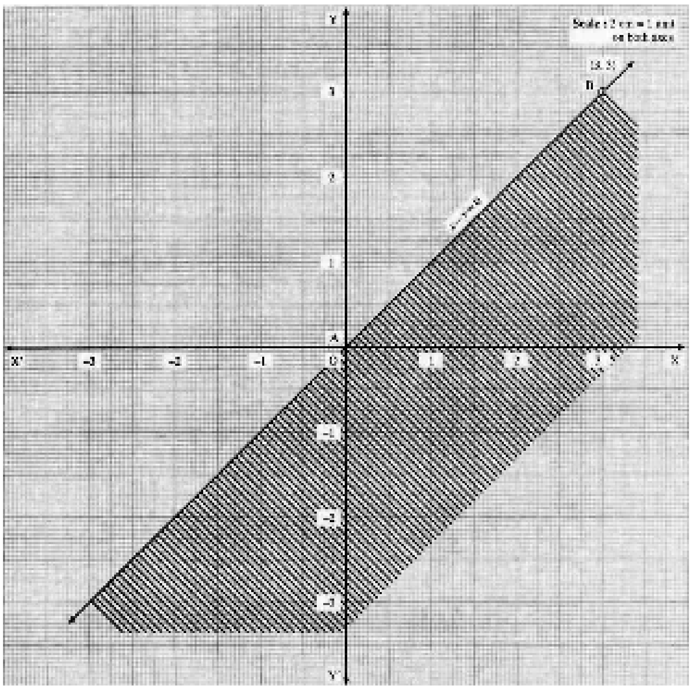
Point	x	y
A	0	0
B	3	3

Two points are A(0, 0) and B (3, 3).

$\therefore$ The graph of this line AB passes through origin and (3, 3).

Choose the point (1, 2) not lying on this line. The coordinates of this point does not satisfy the given inequation.

Therefore shade the half plane below this line. The shaded portion as shown in figure represents the solution set of the given inequation.

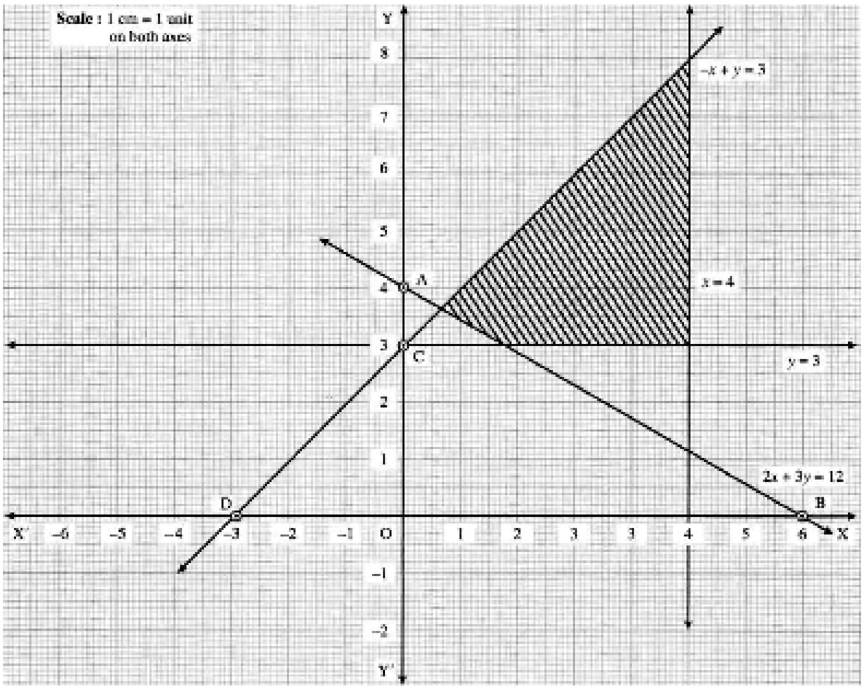


2. $2x - 3y \geq 12$; $-x + y \leq 3$; $x \leq 4$; $y \geq 3$.

To draw the graphs of the given system of linear inequations, we prepare the following table

Inequation	Equations	Points (x, y)				Region
$2x + 3y \geq 12$	$2x + 3y = 12$	x	0	6	A(0, 4)	$2(0) + 3(0) = 0 \not\geq 12$
		y	4	0	B(6, 0)	$\therefore$ Non-Origin side of the line AB
$-x + y \leq 3$	$-x + y = 2$	x	0	-3	C(0, 3)	$0 + 0 = 0 < 3$
		y	3	0	D(-3, 0)	$\therefore$ Origin side of the line CD
$x \leq 4$	$x = 4$	(4, 0)				Parallel to Y-axis, origin side
$y \geq 3$	$y = 3$	(0, 3)				Parallel to X-axis, non-origin side

x = 0 , y = 0 are coordinate axes



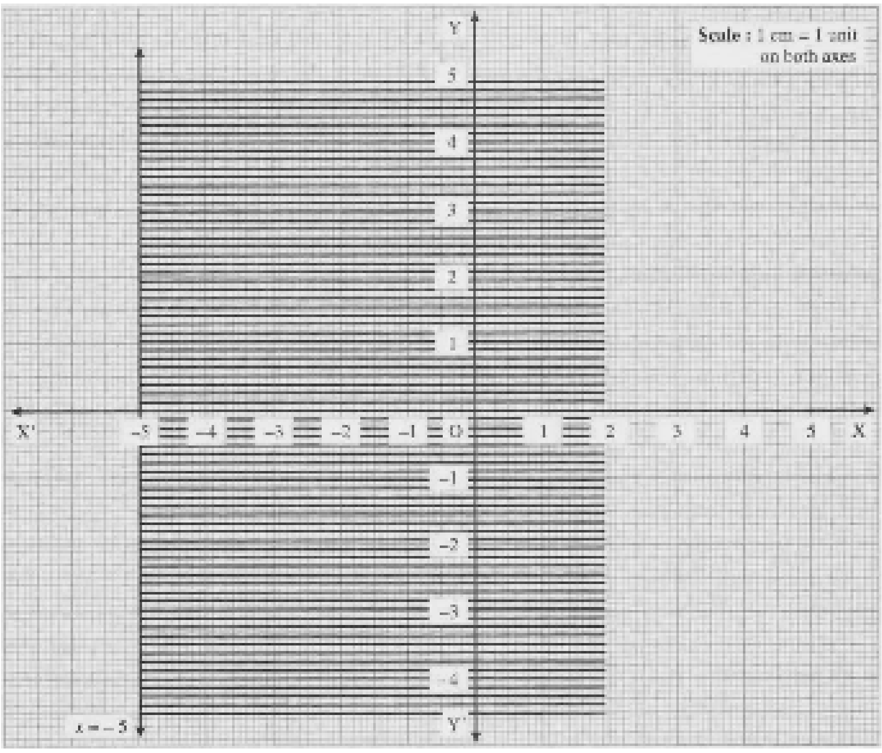
The shaded portion in the graph represents the common region.

3. $-11x - 55 \leq 0 \Rightarrow -11x \leq 55 \Rightarrow -x \leq 55/11$

$\Rightarrow x \geq -5$

$\Rightarrow x = -5$

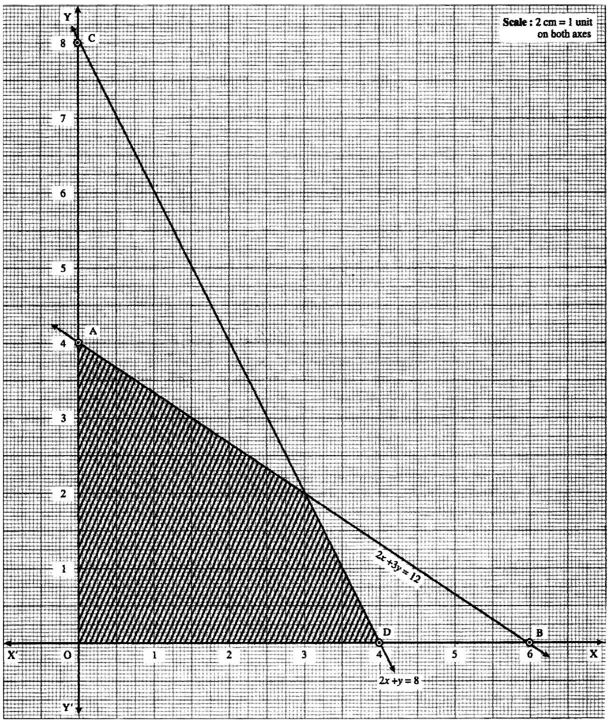
The graph of this line is parallel to Y-axis passing through the point (-5, 0). Therefore shade the half plane right to the line. The shaded portion as shown in the figure represents the solution set of the given inequation.



4. $2x + 3y \leq 12$, $2x + y \leq 8$, $x \geq 0$, $y \geq 0$

To draw the graphs of the given system of linear inequations, we prepare the following table

Inequation	Equation	Points (x, y)				Region
$2x + 3y \leq 12$	$2x + 3y = 12$	x	0	6	A(0, 4)	$2(0) + 3(0) < 12$
		y	4	0	B(6, 0)	$\therefore$ Origin side of the line AB
$2x + y \leq 8$	$2x + y = 8$	x	0	4	C(0, 8)	$2(0) + (0) \leq 8$
		y	8	0	D(4, 0)	$\therefore$ Non-Origin side of the line CD
$x \geq 0, y \geq 0$	$x = 0, y = 0$	-				First Quadrant



The shaded portion in the graph represents the region of the feasible solution for the given system of linear inequations.

5. The inequation is $\dfrac{x + 5}{|x - 3|} < 0$

$\therefore$ Solve $x + 5 > 0$ and $x - 3 < 0$

$\Rightarrow$ $x > -5$ and $x < 3$

OR $x + 5 < 0$ and $x - 3 > 0$

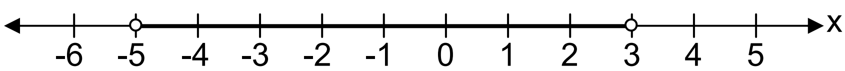
But $x < -5$ and $x > 3$ is not possible, since the value of x cannot be greater than 3 at the same time less than -5.

$\therefore$ The Solution set is $x > -5$ and $x < 3$

i.e., $\{x \mid -5 > x < 3\}$

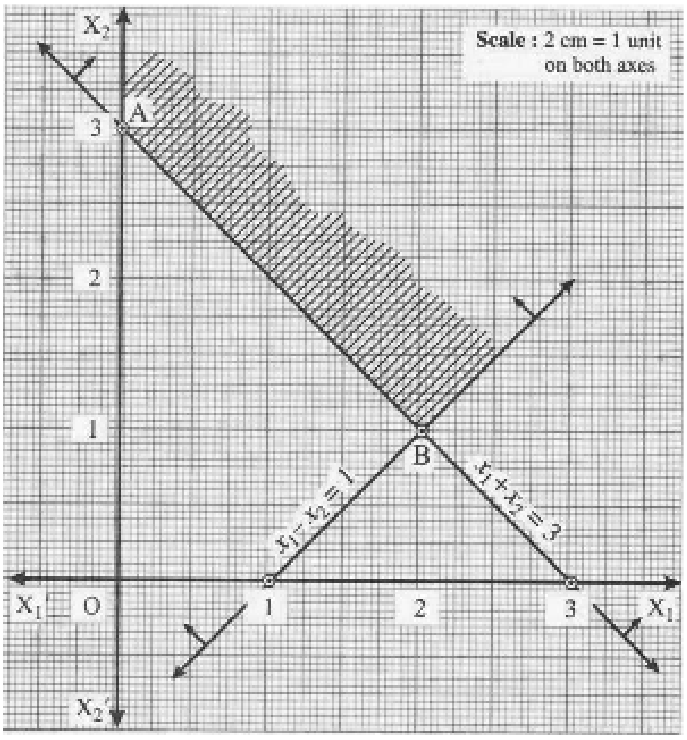
Solution interval : (-5, 3)

Solution graph : It is shown as in figure

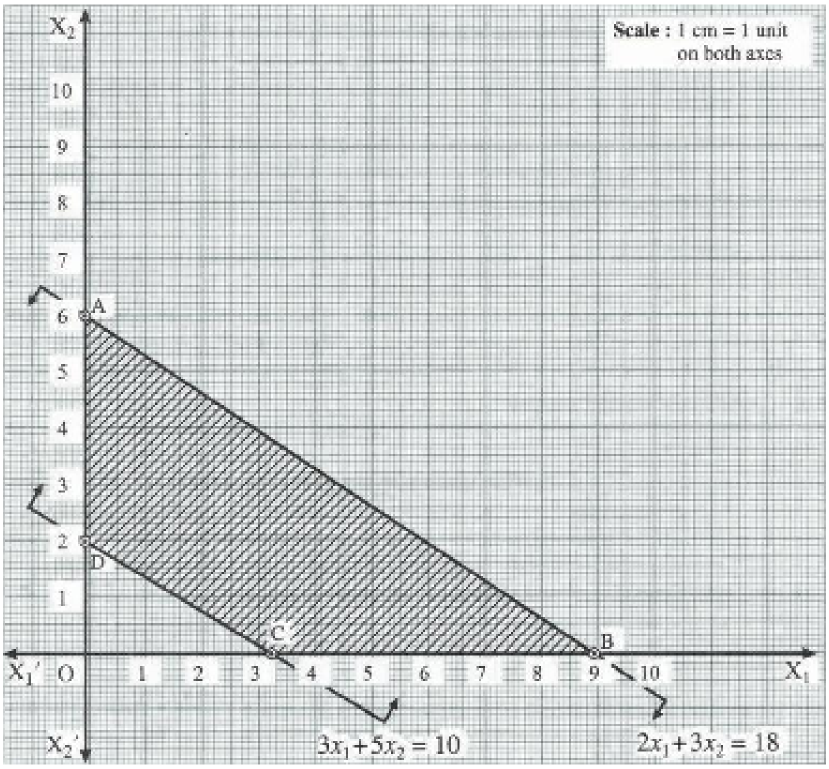


Q.2. Attempt any Four : (3 Marks each) (12)

1. Given : $Z = 3x_1 + 2x_2$.
$x_1 - x_2 \leq 1$, $x_1 + x_2 \geq 3$, $x_1, x_2 \geq 0$
Now, $x_1 - x_2 = 1$
$\therefore$ Two points are (1, 0), (2, 1)
$x_1 + x_2 = 3$
$\therefore$ Two points are (3, 0), (0, 3)
In graph …. AB … is unbounded feasible region This is a convex polygon whose lower vertices are A(0, 3) and B(2, 1). At atleast one of the vertices the value of $Z = 3x_1 + 2x_2$ will be minimum.
At A(0, 3),
Z = 3(0) + 2(3)
= 6
B(2, 1),
Z = 3(2) + 2(1)
= 8
$\therefore$ At A(0, 3) Z is minimum
Hence, the optimum solution is $x_1 = 0$, $x_2 = 3$, $Z_{min} = 6$



2. Given : $Z = 6x_1 + 10x_2$.
$3x_1 + 5x_2 \geq 10$, $2x_1 + 3x_2 \leq 18$, $x_1, x_2 \geq 0$
Now, $3x_1 + 5x_2 = 10$
$\therefore$ Two points are $\left(\frac{10}{3}, 0\right)$ and (0, 2)
$2x_1 + 3x_2 = 18$
$\therefore$ Two points are (9, 0) and (0, 6)



From graph we can see that common feasible region is ABCD with vertices A(0, 6), B(9, 0), $C\left(\frac{10}{3},0\right)$ and D(0, 2)

At A(0, 6), $Z = 6 \times 0 + 10(6) = 60$

B(9, 0), $Z = 6(9) + 10(0) = 54$

$C\left(\frac{10}{3},0\right)$ $Z = 6\left(\frac{10}{3},0\right) + 10(0) = 20$

D(0, 2), $Z = 6(0) + 10(2) = 20$

$\therefore$ at A (0, 6) the value of Z is maximum.

Hence, the unique solution is

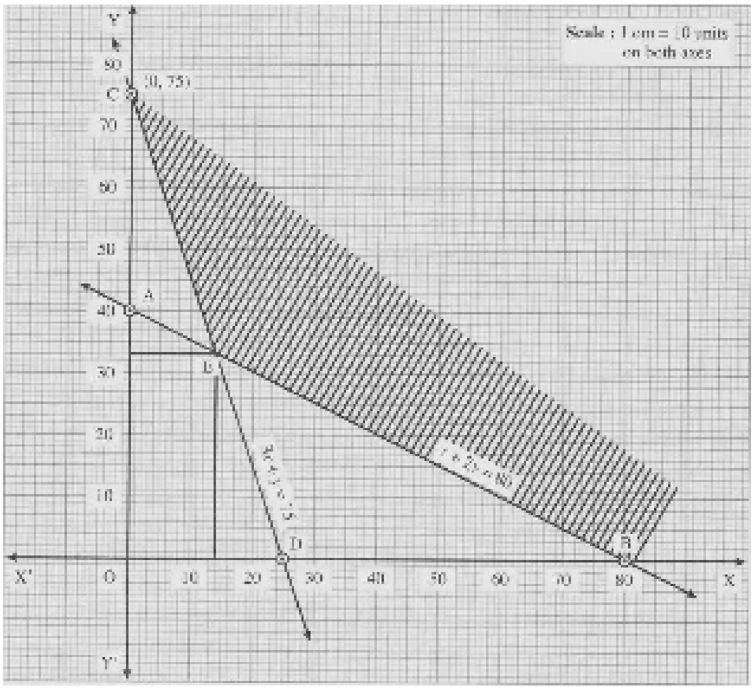
$x_1 = 0$, $x_2 = 6$, $Z_{max} = 60$.

3. Minimize $Z = 4x + 2y$.

$3x + y \geq 27$, $x + y \geq 21$, $x + 2y \geq 30$, $x \geq 0$, $y \geq 0$.

To draw the graph, we prepare the following table

Inequation	Equations	Points (x, y)				Region
$3x + y \geq 27$	$3x + y = 27$	x	1	9	A(1, 12)	$3(0) + 2(0) \ngeq 1$
		y	12	0	B(9, 0)	$\therefore$ Non-Origin side of the line AB
$x + y \geq 21$	$x + y = 21$	x	0	21	C(0, 21)	$0 + 0 \ngeq 21$
		y	21	0	D(21, 0)	$\therefore$ Non-Origin side of the line CD
$x \geq 0, y \geq 0$	$x = 0, y = 0$					First Quadrant



The shaded unbounded portion CGF in the graph represents the graphical solution of the given LPP.

The lower vertices of the unbounded region are C(0, 21), G(12, 9) and F(30, 0) respectively. At any one of these vertices the minimum value of Z is obtained.

Now, Z = 4x + 2y

$\therefore$ at C(0, 21), Z = 4(0) + 2(20) = 40
G(12, 9), Z = 4(12) + 2(9) = 48 + 18 = 66
F(30, 0), Z = 4(30) + 2(0) = 120

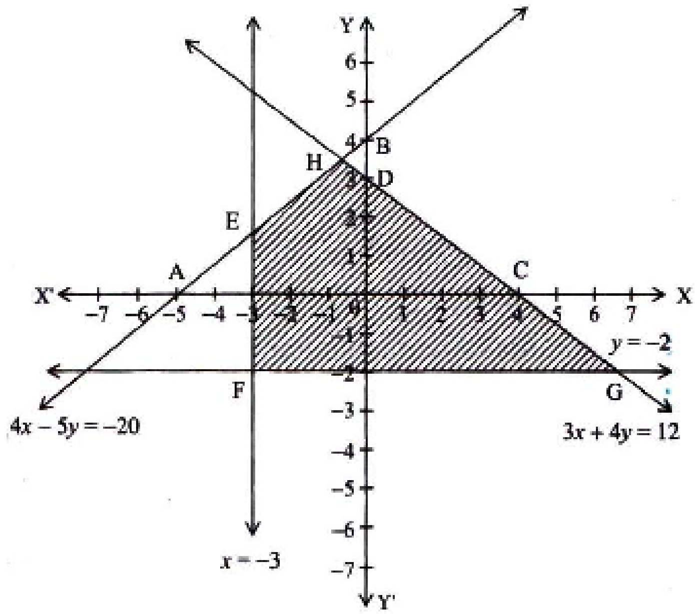
The minimum value of Z is obtained at the point C. Therefore the solution of the given LPP is as follows :

x = 0, y = 20, Z_{min} = 40.

4.

Inequation	Equation	Double Intercept form	Point (x, y)	Region
$x \geq -3$	x = -3	-	-	RHS of line x = -3
$4x - 5y \geq -20$	4x – 5y = -20	$\frac{x}{-5} + \frac{y}{4} = 1$	A(-5, 0), B(0, 4)	4(0) - 5(0) = 0 > -20 $\therefore$ Origin side
$3x + 4y \leq 12$	3x + 4y = 12	$\frac{x}{4} + \frac{y}{3} = 1$	C(4,0), D(0, 3)	3(0) + 4(0) = 0 < 12 $\therefore$ Origin side
$y \geq -2$	y = -2	-	-	Above line y = -3

Shaded region EFGH is the common region.



5. Let x_1 : Number of units of Food F_1
and x_2 : Number of units of Food F_2

Product	Food F_1	Food F_2	Minimum Requirement
Vitamins	200	100	4000
Minerals	1	2	50
Calories	40	30	1500
Cost/unit	₹50	₹75	

Sick person's problem is to determine x_1 and x_2 so as to minimize the total cost

$$Z = 50x_1 + 75x_2$$

Subject to constraints

$$200x_1 + 100x_2 \geq 4000$$
$$x_1 + 2x_2 \geq 50$$
$$40x_1 + 30x_2 \geq 1500$$
$$x_1 \geq 0, \ x_2 \geq 0$$

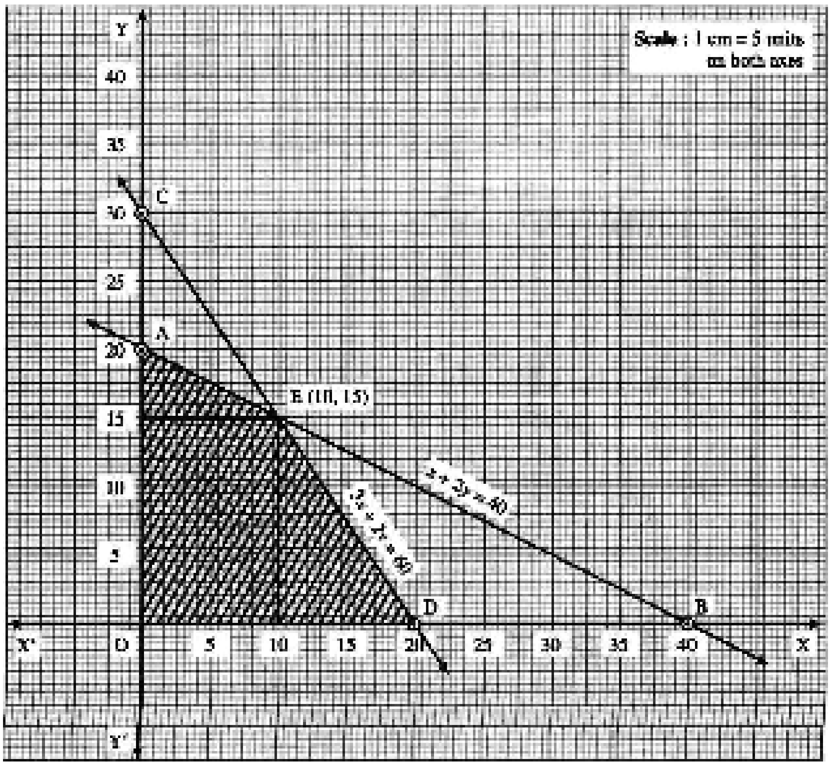
Q.3. Attempt any One : (4 Marks each) (04)

1. Minimize $Z = 60x + 50y$.

$x + 2y \leq 40$, $3x + 2y \leq 60$, $x \geq 0$, $y \geq 0$.

To draw the graph, we prepare the following table :

Inequation	Equations	Points (x, y)				Region
$x + 2y \leq 40$	$x + 2y = 40$	x	0	40	A(0, 20)	$0 + 2(0) < 40$
		y	20	0	B(40, 0)	$\therefore$ Origin side of the line AB
$3x + 2y \leq 60$	$3x + 2y = 60$	x	0	20	C(0, 30)	$3(0) + 2(0) < 60$
		y	30	0	D(20, 0)	$\therefore$ Origin side of the line CD
$x \geq 0, y \geq 0$	$x = 0, y = 0$	-				First quadrant



The shaded portion ODEA in the graph represents the graphical solution of the given LPP. The vertices of ODEA are 0(0, 0), D (20, 0), E (10, 15) and A (0, 20) respectively. At any of these vertices the maximum value of Z is obtained..•. at 0(0, 0), Z = 60 (0) + 50 (0) = 0 D (20, 0), Z = 60(20) + 50(0) = 1200

Now, Z = 60x + 50y

∴ at O(0,0), Z = 60(0) + 50(0) = 0

D(20, 0), Z = 60(20) + 50(0) = 1200

E(10, 15), Z = 60(10) + 50(15) = 600 + 750 = 1350

A(0, 20) Z = 60(0) + 50(20) = 1000

The value of Z is maximum at the point E. Therefore the solution of the given LPP is as follows : x = 10, y = 15 and Z_{max} = 1350.

2. Let x = Number of mixers.

y = Number of food processors.

Since, the number of mixers and food processors cannot be negative, $x \geq 0$, $y \geq 0$.

From the given data, we get the following inequations :

$3x + 3y \leq 36$

$5x + 2y \leq 50$

$2x + 6y \leq 60$

Let Z = Total profit

Profit on selling a mixer and a food processors is ₹2000 and ₹3000 respectively.

∴ The objective function to be maximized is Z = 2000x + 3000y

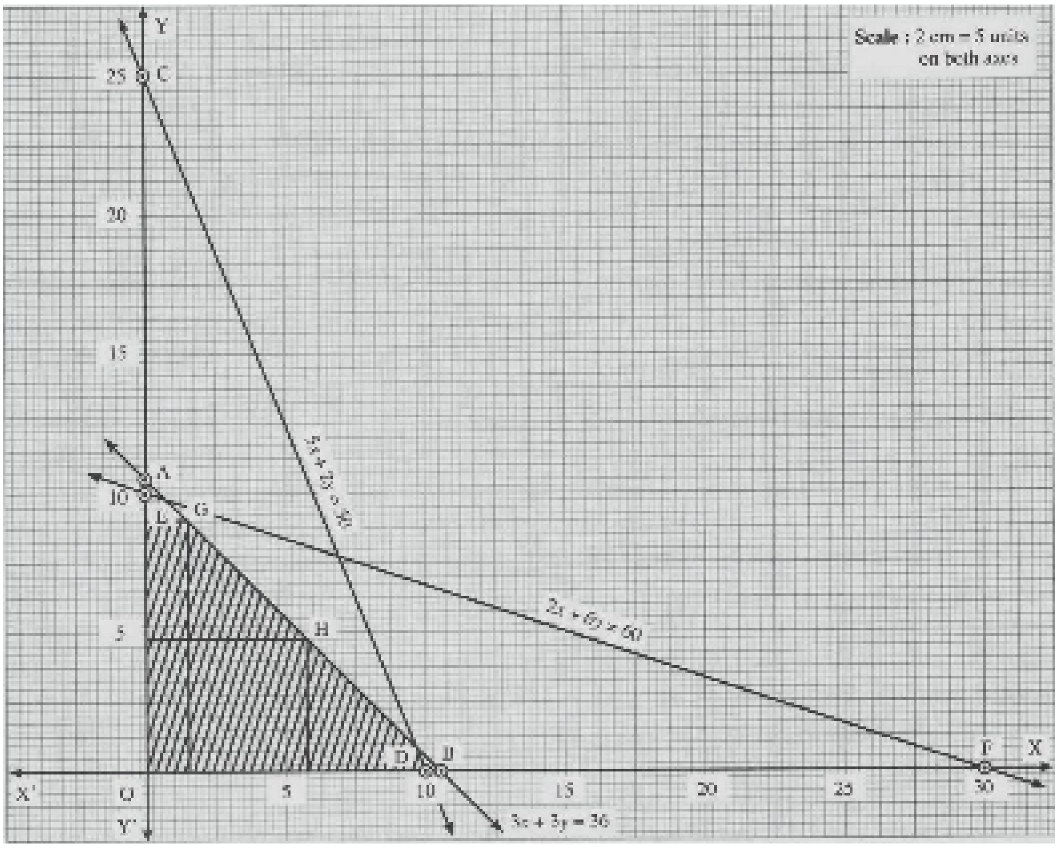
Thus, the LPP formulated is maximize Z = 2000x + 3000y

Subject to $3x + 3y \leq 36$, $5x + 2y \leq 50$

$2x + 6y \leq 60$, $x \geq 0$, $y \geq 0$.

To draw the graph, we prepare the following table :

Inequation	Equations	Points (x, y)				Region
$3x + 3y \leq 36$	3x + 3y = 36	x	0	12	A(0, 12)	3(0) + 3(0)< 36
		y	12	0	B(12, 0)	∴ Origin side of the line AB
$5x + 2y \leq 50$	5x + 2y = 50	x	0	10	C(0, 25)	5(0) + 2(0)< 50
		Y	25	0	D(10, 0)	∴ Origin side of the line CD
$2x + 6y \leq 60$	2x + 6y = 60	x	0	30	E(0, 10)	2(0) + 6(0)< 60
		Y	10	0	F(30, 0)	∴ Origin side of the line EF
$x \geq 0$, $y \geq 0$	x = 0, y = 0	-				First quadrant



From the graph the feasible region is OEGHD. The vertices of this region O(0, 0), E(0, 10), G(3, 9), H$\left(\frac{26}{3},\frac{10}{3}\right)$,

D(10,0). At any one of these vertices, the value of Z is maximum.

Z = 2000x + 3000y

$\therefore$ at O(0, 0), Z = 2000 (0) + 3000 (0) = 0

E(0, 10), Z = 2000 (0) + 3000 (10) = 30000

G(3, 9), Z = 2000 (3) + 3000 (9) = 6000 + 27000 = 33000

H$\left(\frac{26}{3},\frac{10}{3}\right)$, Z = 2000 $\left(\frac{26}{3}\right)$ + 3000 $\left(\frac{10}{3}\right)$ = $\frac{52000+30000}{3}$ = $\frac{82000}{3}$ = 27333.33

D(10,0), Z = 2000 (10) + 3000 (0) = 20000

The value of Z is maximum at the point E(3, 9).

$\therefore$ the solution of the given LPP is x = 3, y = 9, Z_{max} = 33000.

Hence, 3 mixers and 9 food processor should be produced in order to get maximum profit.

3. Let x_1 : Number of passengers traveling by executive class

x_2 : Number of passengers traveling by economy class.

Since, the number of products cannot be negative.

$x_1 \geq 0, x_2 \geq 0$

The airline reserves at least 30 seats for executive class.

$\therefore$ $x_1 \geq 30$

At least 4 times as many passengers prefer economy class than executive class.

$\therefore$ $x_2 \geq 4x_1$

An aeroplane can carry a maximum of 250 passengers.

$\therefore$ $x_1 + x_2 \leq 250$

Objective function :

A profit of ₹ 1500 made on each executive class ticket and ₹ 900 made on each economy class ticket.

$\therefore$ Total profit $Z = 1500x_1 + 900x_2$

Hence, LPP is formulated as follows :

Maximize $Z = 1500x_1 + 900x_2$

Subject to constraints,

$x_1 \geq 30,\ x_2 \geq 4x_1$

$x_1 + x_2 \leq 250$

$x_1 \geq 0,\ x_2 \geq 0.$

Q.4. Solve any Two : (2 Marks each) **(06)**

(1) Given : $n = 15,\ \bar{x} = 25,\ \bar{y} = 18,\ \sigma_x = 3.01,\ \sigma_y = 3.03,\ \sum(x_i - \bar{x})(y_i - \bar{y}) = 122.$

$$\therefore r = \frac{\frac{1}{n}\sum(x_i - \bar{x})(y_i - \bar{y})}{\sigma_x . \sigma_y}$$

$$r = \frac{\frac{1}{5} x\, 122}{3.01 x\, 3.03}$$

$$= \frac{8.1333}{9.1203}$$

$$= 0.8917 \approx 0.89$$

$\therefore r = 0.89$

(2)

	x	**y**	**xy**	**x^2**	**y^2**
	4	5	20	16	25
	2	6	12	4	36
	7	2	14	49	4
	1	7	7	1	49
	5	4	20	25	16
	3	3	9	9	9
	6	1	6	36	1
n = 7	$\Sigma x = 28$	$\Sigma y = 28$	$\Sigma xy = 88$	$\Sigma x^2 = 140$	$\Sigma y^2 = 140$

Correlation Coefficient :

$$\therefore r = \frac{\frac{1}{n}\sum x_i y_i - \bar{x}\bar{y}}{\sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2} . \sqrt{\frac{\sum y_i^2}{n} - (\bar{y})^2}}$$

Putting $n = 7$, $\Sigma x = 28$, $\Sigma y = 28$, $\Sigma xy = 88$, $\Sigma x^2 = 140$, $\Sigma y^2 = 140$ in the formula, we get

$$\therefore r = \frac{\frac{1}{7} x\, 88 - (4)(4)}{\sqrt{\frac{140}{7} - (4)^2} . \sqrt{\frac{140}{7} - (4)^2}}$$

$= \frac{-3.42}{4}$

$= -0.86$

(3) Here, let x_i = rank in Mathematics,

y_i = rank in Physics.

We construct the following table to calculate rank correlation coefficient :

x_i	y_i	$d_i = (x_i - y_i)$	d_i^2
1	1	0	0
2	10	- 8	64
3	3	0	0
4	4	0	0
5	5	0	0
6	7	-1	1
7	2	05	25
8	6	02	4
9	8	01	1
10	9	01	1
Total	n = 10	-	$\sum d_i^2 = 96$

Rank Correlation coefficient :

$R = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$,

Putting n = 10, $\sum d_i^2$ = 96 in the formula, we get

$R = 1 - \frac{6 \times 96}{10(10^2-1)}$

$R = 1 - \frac{576}{10(100-1)}$

$= 1 - \frac{576}{10 \times 99}$

$= 1 - \frac{576}{10 \times 99}$

$= 1 – 0.582 = 0.418$

$\therefore R = 0.42$

(4) We construct the following table for computing rank correlation coefficient :

R_x	R_y	$d_i = (R_x – R_y)$	d_i^2
1	6	-5	25
2	3	-1	1
3	2	1	1
4	1	3	9
5	4	1	1
6	5	1	1
n = 6	-	-	$\sum d_i^2 = 38$

Rank Correlation Coefficient :

$R = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$

Putting n = 6, $\sum d_i^2 = 38$ in the formula, we get

$$1 - \frac{6 \times 38}{6\left(6^2 - 1\right)}$$

$$R = 1 - \frac{38}{35}$$

$= 1 - 1.0857$

$= -0.0857$

$\therefore R = -0.0857$

Q.5. Solve any Four : (3 Marks each) (12)

(1) $r = 0.4,\ \sum(x_i - \bar{x})(y_i - \bar{y}) = 108,\ \sigma_y = 3$ and $\sum(x_i - \bar{x})^2 = 900$

$$\text{Now, } r = \frac{\frac{1}{n}\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sigma_x \cdot \sigma_y}$$

$$= \frac{\frac{1}{n}\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n}} \times \sigma_y}$$

Putting $r = 0.4,\ \sum(x_i - \bar{x})(y_i - \bar{y}) = 108,\ \sigma_y = 3$ and $\sum(x_i - \bar{x})^2 = 900$ in the formula, we get

$$\therefore 0.4 = \frac{\frac{1}{n} \times 108}{\sqrt{\frac{900}{n}} \times 3}$$

$$\therefore 0.4 = \frac{36}{n \times \sqrt{\frac{900}{n}}}$$

$$\therefore 0.4 = \frac{36}{\sqrt{n} \times \sqrt{900}}$$

$$\therefore 0.16 = \frac{1296}{n \times 900} \quad \text{.......... (Taking square of both sides)}$$

$\therefore 0.16 \times n \times 900 = 1296$

$\therefore 144n = 1296$

$$\therefore n = \frac{1296}{144}$$

$\therefore n = 9$

(2) We construct the following table for computing correlation coefficient :

	x_i	y_i	$x_i\, y_i$	x_i^2	y_i^2
	9	19	171	81	361
	7	17	119	49	289
	6	16	96	36	256
	8	18	144	64	324
	9	19	171	81	361
	6	16	96	36	256
	7	17	119	49	289
n = 7	$\sum x_i = 52$	$\sum y_i = 122$	$\sum x_i y_i = 916$	$\sum x_i^2 = 396$	$\sum y_i^2 = 2136$

$\bar{x} = \dfrac{\sum x_i}{n} = \dfrac{52}{7} = 7.43$

$\bar{y} = \dfrac{\sum y_i}{n} = \dfrac{122}{7} = 17.43$

Correlation Coefficient :

$$\therefore r = \frac{\frac{1}{n}\sum x_i y_i - \bar{x}\bar{y}}{\sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2} \cdot \sqrt{\frac{\sum y_i^2}{n} - (\bar{y})^2}}$$

Putting n = 7, $\sum x_i y_i = 916$, $\bar{x} = 7.43$, $\bar{y} = 17.43$, $\sum x_i^2 = 396$, $\sum y_i^2 = 2136$ in the formula, we get

$$\therefore r = \frac{\frac{1}{7} \text{ x } 916 - (7.43)(17.43)}{\sqrt{\frac{396}{7} - (7.43)^2} \cdot \sqrt{\frac{2136}{7} - (17.43)^2}}$$

$$= \frac{130.857 - 129.505}{\sqrt{56.571 - 55.205} \cdot \sqrt{305.143 - 303.805}}$$

$$= \frac{1.352}{\sqrt{1.366} \cdot \sqrt{1.338}}$$

$$= \frac{1.352}{1.169 \text{ x } 1.157}$$

$$= \frac{1.352}{1.352}$$

$= 1$

$\therefore r = 1$

(3) Given : $R = \dfrac{2}{3}$, $\sum d_i^2 = 55$

Now, $R = 1 - \dfrac{6\sum d_i^2}{n(n^2 - 1)}$

$\therefore \dfrac{2}{3} = 1 - \dfrac{6 \text{ x } 55}{n(n^2 - 1)}$

$\therefore \dfrac{330}{n(n^2 - 1)} = 1 - \dfrac{2}{3}$

$\therefore \dfrac{330}{n(n^2 - 1)} = \dfrac{1}{3}$

$\therefore 330 \text{ x } 3 = n(n^2 - 1)$

$\therefore n(n^2 - 1) = 990$

$\therefore n(n^2 - 1) = 10 \text{ x } 99$

$\therefore n(n^2 - 1) = 10 \text{ x } (10^2 - 1)$

$\therefore n = 10$

(4) We are given that, n = 100, $\bar{x}$ = 62, $\bar{y}$ = 53, σ_x = 10, σ_y = 12, $\sum(x_i-\bar{x})(y_i-\bar{y})$ = 8,000

For finding correlation coefficient we required

Cov (X, Y) and σ_x and σ_y

Cov (X, Y) $= \frac{1}{n}\sum(x_i-\bar{x})(y_i-\bar{y})$

$= \frac{8000}{100}$

= 80

Cov (X, Y) $= \frac{Cov(X,Y)}{\sigma_x\ \sigma_y}$

$= \frac{80}{10 \times 12}$

= 0.67

(5) Giving the ranks to the contestants according to their marks, we have following table :

Contestants	**Marks by A**	**Marks by B**	**Ranks of A**	**Ranks of B**	**d = x_i - y_i**	**d_i^2**
			(x_i)	**(y_i)**		
1	81	75	1	2	-1	1
2	72	56	2	4	-2	4
3	60	42	3	5	-2	4
4	33	15	6	8	-2	4
5	29	30	7	6	1	1
6	11	20	8	7	1	1
7	56	60	4	3	1	1
8	42	80	5	1	4	16
Total						32

From Table $\sum d_i^2$ = 32

$\therefore$ R = 1 - $\frac{6\sum d_i^2}{n(n^2-1)}$

= 1 - $\frac{6 \times 32}{8(64-1)}$

= 0.6190

There is considerable agreement between the two judges with respect to elocution.

Q.6. Solve any Three : (4 Marks each) **(12)**

(1) Given r = 0.6, $\sum d_i^2$ = 66

$\therefore$ R = 1 - $\frac{6\sum d_i^2}{n(n^2-1)}$

$\therefore$ 0.6 = 1 - $\frac{6\sum d_i^2}{n(n^2-1)}$

$\therefore$ 0.6 = 1 - $\frac{6 \times 66}{n(n^2-1)}$

$\therefore \dfrac{6 \times 66}{n(n^2 - 1)} = 0.4$

$\therefore n(n^2 - 1) = \dfrac{6 \times 66}{0.4} = 990$

$\therefore (n - 1)(n + 1) = 990 = 9 \times 10 \times 11$

$\therefore n = 10$

(2) For finding correlation coefficient we need to find covariance and standard deviations of X and Y. We construct following table.

x_i	y_i	x_i^2	y_i^2	x_iy_i
1	12	1	144	12
2	11	4	121	22
3	13	9	169	39
4	15	16	225	60
5	14	25	196	70
6	17	36	289	102
7	16	49	256	112
8	19	64	361	152
9	18	81	324	162
45	**135**	**285**	**2085**	**731**

From table we have,

$\sum x_i = 45,\ \sum y_i = 135,\ \sum x_i^2 = 285,\ \sum y_i^2 = 2085,\ \sum x_iy_i = 731$

$\therefore \bar{x} = \dfrac{\sum x_i}{n} = \dfrac{45}{9} = 5$

$\therefore \bar{y} = \dfrac{\sum y_i}{n} = \dfrac{135}{9} = 15$

$$\text{Cov}(X, Y) = \frac{1}{n}\sum_{i=1}^{n} x_iy_i - \bar{x}\bar{y}$$

$$= \frac{731}{9} - 5 \times 15$$

$$= 81.22 - 75$$

$$= 6.22$$

$$\sigma_x^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{285}{9} - (5)^2$$

$$= 31.66 - 25 = 6.66$$

$$\sigma_y^2 = \frac{\sum y_i^2}{n} - (\bar{y})^2$$

$$= \frac{2085}{9} - (15)^2$$

$$= 231.66 - 225$$

$$= 6.66$$

$$r = \frac{\text{Cov}(X, Y)}{\sigma_x . \sigma_y}$$

$= \frac{6.22}{\sqrt{6.66}\sqrt{6.66}}$

$= \frac{6.22}{6.66}$

$= 0.93$

There is high degree positive correlation between X and Y.

(3) Given : $r_{xy} = 0.8$, $\sum x_i y_i = 60$, $\sigma_y = 2.5$ and $\sum x_i^2 = 90$.

$x_i = \left(x_i - \bar{x}\right)$, $y_i = \left(y_i - \bar{y}\right)$

$\therefore \sum x_i y_i = \sum \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)$

$\therefore \sum x_i^2 = \sum \left(x_i - \bar{x}\right)^2$

Now, $r = \frac{\frac{1}{n}\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sigma_x . \sigma_y}$

Here, $r_{xy} = 0.8$, $\sum \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) = 60$, $\sigma_y = 2.5$

$\sigma_x = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n}} = \sqrt{\frac{90}{n}}$

Putting these values in the formula, we get

$0.8 = \frac{\frac{60}{n}}{\sqrt{\frac{90}{n}} \times 2.50}$

$\therefore 0.8 = \frac{60}{n \times \sqrt{\frac{90}{n}} \times 2.50}$

$\therefore 0.8 = \frac{24}{\sqrt{n} . \sqrt{90}}$

$\therefore 0.64 = \frac{576}{n \times 90}$ .... (Taking square of both sides)

$\therefore 0.64 \times n \times 90 = 576$

$\therefore n \times 57.6 = 576$

$\therefore n = \frac{576}{57.6} = 10$